Investigation 4: Pattern Recognition

Example 1

What rule seems to describe this sequence? Find the next two terms in the sequence.

3, 6, 12, 24, ____, ____, ...

Since $3 \times 2 = 6$, $6 \times 2 = 12$, and $12 \times 2 = 24$, we **multiply by 2** to find the next terms. Thus, the fifth term of the sequence is 24×2, or **48,** and the sixth term is 48×2, or **96.**

A sequence that counts up (adds) or counts down (subtracts) by the same amount is called an **arithmetic sequence.** A sequence that increases by multiplying by the same number or decreases by dividing by the same number is called a **geometric sequence.**

In problems **1–4** below, decide whether the sequence is arithmetic or geometric. Then write the next three terms.

1. 43, 49, 55, 61, ____, ____, ____, ...
2. 2, 4, 8, 16, ____, ____, ____, ...
3. 50, 48, 46, 44, ____, ____, ____, ...
4. 2, 6, 18, 54, ____, ____, ____, ...
5. Samantha has saved $55. Each month she plans to add $8 to her savings. If she does not spend any of the money she saves, how much will Samantha have after one month? Two months? Three months? What kind of sequence are we making?
6. The fruit fly population in an experiment doubled every week. If the experiment began with 50 fruit flies, what was the population after one week? Two weeks? Three weeks? What kind of sequence are we making?

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7. Assuming the following sequence repeats after every three terms, write the next three terms:

   4, 5, 9, 4, 5, 9, 4, ____, ____, ____, ...

8. Assuming the following sequence repeats after every four terms, write the next three terms:

   5, 2, 3, 6, 5, 2, ____, ____, ____, ...

9. Assuming the following sequence repeats after every four terms, write the next three terms:

   B, U, L, B, ____, ____, ____, ...

10. Assuming the sequence in problem **9** repeats after every three terms, write the next three terms.

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Find the next three terms in these sequences:

16. 1, 4, 9, 16, 25, ____, ____, ____, ...
17. 2, 3, 5, 8, 12, ____, ____, ____, ...

Example 5

Suppose the first two terms of a sequence are 3 and 4 and that we always get the next term by adding the previous two terms together. The third term would be $3 + 4 = 7$. Find the fourth, fifth, and sixth terms of the sequence:

3, 4, 7, ____, ____, ____, ...

We find each term by adding the two preceding terms. Three and 4 were added to get the third term, 7. Now we add 4 and 7 to find the fourth term, 11.

3, 4, 7, **11**, ____, ____, ...

We continue adding the two preceding terms. The sum of 7 and 11 is 18. The sum of 11 and 18 is 29.

3, 4, 7, **11**, **18**, **29**, ...

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There are many types of patterns that sequences can follow. In the next example, we look at another kind of pattern.

Example 3

What pattern does this sequence appear to follow?

1, 0, 1, 0, 0, 1, 0, 0, 0, 1, …

This sequence of 0s and 1s has 1s separated by an increasing number of 0s. First, one 0 separates 1s; then two 0s; then three 0s. It is reasonable to predict that there will be four more 0s before the next 1 that appears in the sequence.

1, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, …
(1, 2, 3, 4 zeros)

We can predict that there will then be five more 0s between 1s, six more 0s, and so on.

For problems **11–15,** describe the pattern that the sequence appears to follow. Then write the next few terms that seem to fit the pattern.

11. 1, 1, 2, 2, 3, 3, …

12. 0, 2, 0, 4, 0, 6, 0, …

13. A, B, D, E, G, H, …

14. T, ⊣, ⊥, ⊢, T, …

15. 1, 2, 1, 2, 3, 1, 2, 3, 4, …

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